

Natural Doublet-Triplet Splitting in $SU(N) \times U(1)$

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Abstract

It is shown that natural doublet-triplet splitting can be achieved in a relatively simple way in supersymmetric $SU(N) \times U(1)$ models with $N > 5$.

Much more attention has been paid to $SO(10)$ unification than to unification based on $SU(N)$ with $N > 5$. Part of the appeal of $SO(10)$ is that the observed quarks and leptons fit perfectly into spinor multiplets. As pointed out long ago, however, there is also a simple explanation of the observed fermion spectrum in the context of large unitary groups. Any anomaly-free set of antisymmetric tensor multiplets of fermions in $SU(N)$, with $N > 5$, will decompose under the Standard Model subgroup into families with the observed quantum numbers [1]. Interestingly, in such $SU(N)$ models, the different families can come from tensor multiplets of different size, unlike the situation in $SU(5)$ and $SO(10)$, where the families arise from having three copies of the same multiplets ($3 \times (\mathbf{10} + \mathbf{\bar{5}})$ or $3 \times (\mathbf{16})$, respectively). This feature may be relevant to explaining the observed mass hierarchy among the families [2].

One problem with $SU(N)$ unification, however, is achieving doublet-triplet splitting [3] in a way that is both technically natural and economical. In $SU(5)$, the only technically natural doublet-triplet splitting is by the missing partner mechanism [4], but it requires Higgs fields in rather large tensor multiplets, specifically $H_{[\delta\epsilon]}^{[\alpha\beta\gamma]} = \mathbf{50}$ and $H_{[\delta\epsilon]}^{[\alpha\beta]} = \mathbf{75}$. This can be generalized to $SU(N)$, $N > 5$, but has the same drawback there. The “sliding singlet” mechanism can be made to work in $SU(N)$ models with $N > 5$ [5], but it requires the Higgs superpotential to have a very special form [6]. There is also the so-called GIFT mechanism [7], which can be implemented in models with unitary groups larger than $SU(5)$.

As is well known, the missing partner mechanism can be implemented in an extremely economical way in supersymmetric “flipped $SU(5)$ ” [8] (the group of which is really $SU(5) \times U(1)$). In this paper, we show that this can be generalized to supersymmetric $SU(N) \times U(1)$.

The possibility of a “flipped” breaking of $SU(5) \times U(1)$ to the Standard Model in addition to the “Georgi-Glashow” breaking can be understood group-theoretically as arising from the fact that $SU(5) \times U(1)$ is embeddable in $SO(10)$ in two distinct ways. Even if the $SU(5) \times U(1)$ gauge group is not actually embedded in an $SO(10)$ gauge group, the flipped breaking requires that the $U(1)$ charges be “ $SO(10)$ -like”, i.e. that the fermion multiplets $\mathbf{10}$, $\mathbf{\bar{5}}$, and $\mathbf{1}$ have $U(1)$ charges 1, -3, and 5, just as though they came from a spinor of $SO(10)$, and similarly for the charges of Higgs multiplets.

In a realistic $SU(N) \times U(1)$ model, with $N > 5$, however, the fermion

multiplets are not embeddable in an $SO(10)$ (nor, in most cases, in any larger orthogonal group). Nevertheless, in many simple $SU(N) \times U(1)$ models it happens that the $U(1)$ charges of the quark/lepton multiplets are forced to be “ $SO(10)$ -like” by anomaly cancellation. That is enough to allow for flipped breaking of $SU(N) \times U(1)$ down to the Standard Model. With such breaking, as will be seen, an economical implementation of the missing partner mechanism is possible. Unlike the case of $SU(5) \times U(1)$, however, it seems to require that some of the known quarks and leptons obtain mass from higher-dimension effective Yukawa operators.

We shall briefly review how the missing partner mechanism works in flipped $SU(5) \times U(1)$. Then we shall show explicitly how it can be generalized to $SU(6) \times U(1)$ and $SU(7) \times U(1)$ models. The further generalization to larger unitary groups is relatively straightforward.

In the simplest supersymmetric flipped $SU(5)$ model, the quarks and leptons of a family are in $\mathbf{10}^1 + \bar{\mathbf{5}}^{-3} + \mathbf{1}^5$ of $SU(5) \times U(1)_X$ and the Higgs multiplets include $\mathbf{10}_H^1 + \bar{\mathbf{10}}_H^{-1} + \mathbf{5}_H^{-2} + \bar{\mathbf{5}}_H^2$. The breaking of $SU(5) \times U(1)_X \rightarrow SU(3)_c \times SU(2)_c \times U(1)_Y$ is done by $\langle H^{12} \rangle \in \mathbf{10}_H^1$ and $\langle H_{12} \rangle \in \bar{\mathbf{10}}_H^{-1}$. This leaves unbroken $Y/2 = \frac{1}{5}(-Y_5/2 + X)$, where $Y_5/2$ is the $SU(5)$ generator $diag(\frac{1}{2}, \frac{1}{2}, -\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3})$. The Higgs superpotential is assumed to have the couplings $\mathbf{10}_H^1 \mathbf{10}_H^1 \mathbf{5}_H^{-2} + \bar{\mathbf{10}}_H^{-1} \bar{\mathbf{10}}_H^{-1} \bar{\mathbf{5}}_H^2$, but not the coupling $\mathbf{5}_H^{-2} \bar{\mathbf{5}}_H^2$. Then the superheavy masses of the Higgs(ino) doublets and triplets can be represented as follows:

$$\begin{array}{ccccccc}
\mathbf{5}_H^{-2} & & \mathbf{10}_H^1 & & \bar{\mathbf{10}}_H^{-1} & & \bar{\mathbf{5}}_H^2 \\
\hline
H^i & & \langle H^{12} \rangle & & \langle H_{12} \rangle & & \\
H^a & \longleftrightarrow & \epsilon_{abc} H^{bc} & & \epsilon^{abc} H_{bc} & \longleftrightarrow & H_a
\end{array} \tag{1}$$

Table I.

where $i, j = 1, 2$ are $SU(2)_L$ indices, and $a, b, c = 3, 4, 5$ are $SU(3)_c$ indices. The arrows connecting two multiplets represent superheavy Dirac masses, in this case produced by the couplings $\mathbf{10}_H^1 \mathbf{10}_H^1 \mathbf{5}_H^{-2} + \bar{\mathbf{10}}_H^{-1} \bar{\mathbf{10}}_H^{-1} \bar{\mathbf{5}}_H^2$, which contain $\epsilon_{12abc} \langle H^{12} \rangle H^{bc} H^a + \epsilon^{12abc} \langle H_{12} \rangle H_{bc} H_a$. The square brackets around H^{ia} and H_{ia} in Table I represent the fact that these multiplets are eaten by the Higgs mechanism when H^{12} and H_{12} obtain vacuum expectation values (VEVs). One sees that the doublet fields H^i and H_i do not acquire mass as

they have no weak-doublet, color-singlet “partners” in the $\mathbf{10}_H + \overline{\mathbf{10}}_H$. These are the doublets of the MSSM and couple to the quark and lepton multiplets through the usual terms $(\mathbf{10}^1 \mathbf{10}^1) \mathbf{5}_H^{-2} + (\mathbf{10}^1 \overline{\mathbf{5}}^{-3}) \overline{\mathbf{5}}_H^2 + (\overline{\mathbf{5}}^{-3} \mathbf{1}^5) \mathbf{5}_H^{-2}$.

Let us now see how this mechanism can be generalized to the gauge groups $SU(6) \times U(1)$. The details which we now present are more involved than in flipped $SU(5)$, simply because the group has higher rank and there is another step of symmetry breaking required, and also because the multiplets of $SU(6)$ contain more Standard Model fields, which have to be kept track of. This will be even more the case when we turn to $SU(7) \times U(1)$. But the essence of what is going on is the same as in flipped $SU(5)$, and is basically simple: the missing-partner mechanism of flipped $SU(5)$ is being embedded in larger unitary groups. The matter (quark and lepton) content in each case is essentially minimal, and the structure of the Higgs superpotentials (in Eqs. (2) and (4)) is simple. We now go through the details carefully to demonstrate that, indeed, the mechanism can be implemented in a realistic way and no obstruction arises for these larger groups.

A family in $SU(6)$ consists of $\psi^{AB} + \psi_A + \psi_B = \mathbf{15} + \overline{\mathbf{6}} + \overline{\mathbf{6}}$. (Capital letters $A, B = 1, 2, \dots, 6$ denote $SU(6)$ indices.) Up to an overall normalization, there is only one way to assign $U(1)$ charges to the quarks and leptons in a family-independent way that is consistent with anomaly cancellation, namely $\mathbf{15}^0 + \overline{\mathbf{6}}^1 + \overline{\mathbf{6}}^{-1}$. (The contribution to the anomalies of the Higgsinos would vanish, if the Higgs multiplets are in conjugate pairs under the gauge group as would generally be required to avoid D-term breaking of supersymmetry.) The $U(1)$ charges of the Higgs superfields are determined by their Yukawa couplings to the quark and lepton supermultiplets. Higgs superfields are needed in the following multiplets: $\mathbf{15}_H^0 + \overline{\mathbf{6}}_H^1 + \overline{\mathbf{6}}_H^{-1}$ plus their conjugate multiplets.

The breaking to the Standard Model group is done by the vacuum expectation values $\langle H_6^{(-1)} \rangle \in \overline{\mathbf{6}}_H^{-1}$ and $\langle H^{12} \rangle \in \mathbf{15}_H^0$ and the corresponding vacuum expectation values in the conjugate Higgs multiplets. (We use the superscripts (± 1) to distinguish the two anti-fundamental Higgs multiplets based on their $U(1)$ charge.) The first of these VEVs breaks $SU(6) \times U(1) \rightarrow SU(5) \times U(1)_X$. If T_6 is the $SU(6)$ generator $diag(1, 1, 1, 1, 1, -5)$ and T_1 is the generator of the original $U(1)$ as normalized above, then the generator of the unbroken $U(1)_X$ is given by $X = \frac{1}{2}(T_6 + 5T_1)$. This is clear, since $H_6^{(-1)}$ has $T_6 = 5$ and $T_1 = -1$. The field $H^{12} \in \mathbf{15}_H^0$ then does the usual “flipped”

breaking of $SU(5) \times U(1)_X$ down to the Standard Model group. One can see that it has the correct $SU(5) \times (1)_X$ charges to do this. It has $T_6 = 2$, $T_1 = 0$, and thus $X = 1$. It also has $Y_5/2 = 1$, where $Y_5/2$ is the $SU(6)$ generator $\text{diag}(\frac{1}{2}, \frac{1}{2}, -\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, 0)$, which is also a generator of the $SU(5)$ subgroup. Therefore, its VEV breaks $SU(5) \times U(1)_X$ to the Standard Model group, with $Y/2 = \frac{1}{5}(-Y_5/2 + X) = \frac{1}{5}(-Y_5/2 + \frac{1}{2}T_6 + \frac{5}{2}T_1)$.

The decomposition of the quarks-lepton and Higgs multiplets is given in Table II.

$SU(6)^{T_1}$	$SU(5)^{T_6, T_1}$	$SU(5)^X$	
$\psi^{AB} = \mathbf{15}^0$	$\psi^{\alpha\beta} = \mathbf{10}^{2,0}$	$\mathbf{10}^1$	<i>light</i>
	$\psi^{\alpha 6} = \mathbf{5}^{-4,0}$	$\mathbf{5}^{-2}$	<i>SUPERHEAVY</i>
$\psi_A^{(1)} = \bar{\mathbf{6}}^1$	$\psi_\alpha^{(1)} = \bar{\mathbf{5}}^{-1,1}$	$\bar{\mathbf{5}}^2$	<i>SUPERHEAVY</i>
	$\psi_6^{(1)} = \mathbf{1}^{5,1}$	$\mathbf{1}^5$	<i>light</i>
$\psi_A^{(-1)} = \bar{\mathbf{6}}^{-1}$	$\psi_\alpha^{(-1)} = \bar{\mathbf{5}}^{-1,-1}$	$\bar{\mathbf{5}}^{-3}$	<i>light</i>
	$\psi_6^{(-1)} = \mathbf{1}^{5,-1}$	$\mathbf{1}^0$	<i>SUPERHEAVY</i>
$H^{AB} = \mathbf{15}_H^0$	$H^{\alpha\beta} = \mathbf{10}_H^{2,0}$	$\mathbf{10}_H^1$	$\langle H^{12} \rangle \sim M_G$
	$H^{\alpha 6} = \mathbf{5}_H^{-4,0}$	$\mathbf{5}_H^{-2}$	$\langle H^{26} \rangle \sim M_W$
$H_A^{(1)} = \bar{\mathbf{6}}_H^1$	$H_\alpha^{(1)} = \bar{\mathbf{5}}_H^{-1,1}$	$\bar{\mathbf{5}}_H^2$	
	$H_6^{(1)} = \mathbf{1}_H^{5,1}$	$\mathbf{1}_H^5$	
$H_A^{(-1)} = \bar{\mathbf{6}}_H^{-1}$	$H_\alpha^{(-1)} = \bar{\mathbf{5}}_H^{-1,-1}$	$\bar{\mathbf{5}}_H^{-3}$	$\langle H_2^{(-1)} \rangle \sim M_W$
	$H_6^{(-1)} = \mathbf{1}_H^{5,-1}$	$\mathbf{1}_H^0$	$\langle H_6^{(-1)} \rangle \sim M_G$

Table II.

Note that the X charges are exactly those that would result if $SU(5) \times U(1)_X$ were embedded in an $SO(10)$, even though obviously $SU(6) \times U(1)$ is not a subgroup of $SO(10)$. This can be understood in two ways: (1) in terms of the structure of E_6 , or (2) as the result of anomaly cancellation. The argument based on E_6 is as follows. E_6 contains the subgroups $E_6 \supset SU(6) \times SU(2) \supset SU(6) \times U(1)$ under which $\mathbf{27} \longrightarrow (\mathbf{15}, \mathbf{1}) + (\bar{\mathbf{6}}, \mathbf{2}) \longrightarrow \mathbf{15}^0 + \bar{\mathbf{6}}^1 + \bar{\mathbf{6}}^{-1}$. Note that these $SU(6) \times U(1)$ charges assignments are just those assumed in Table II. But also E_6 contains the subgroups $E_6 \supset SO(10) \supset SU(5) \times U(1)_X$. The fact that the flipped $SU(5) \times U(1)_X$ is a subgroup of the $SU(6) \times U(1)$ in Table II can therefore be understood as a consequence of the group theory of E_6 . This explanation is special to $SU(6) \times U(1)$, however, and does not generalize to $SU(N) \times U(1)$ with $N > 6$.

The anomaly cancellation argument is more general. The point is that if $SU(N) \times U(1)$ is broken to $SU(5) \times U(1)_X$ and the light families end up as $\mathbf{10} + \mathbf{\bar{5}} + \mathbf{1}$ then X has to have the “ $SO(10)$ -like” assignments $\mathbf{10}^1 + \mathbf{\bar{5}}^{-3} + \mathbf{1}^5$ because of anomaly freedom. (This is so, even if each light family contains several extra $SU(5)$ singlets.) This is the crucial point that makes it possible to have “flipped” breaking and to implement the missing partner mechanism in an economical way in $SU(N) \times U(1)$ models, even if they have particle content that cannot be understood in terms of orthogonal or exceptional groups.

In the $SU(6) \times U(1)$ model, the “flipped” breaking of the $SU(5) \times U(1)_X$ subgroup down to the Standard Model group is done (as in ordinary flipped $SU(5)$) by $\langle H^{12} \rangle$ and $\langle H_{12} \rangle$. As shown in Table II, these are contained in the $\mathbf{15}^0 + \mathbf{\bar{15}}^0$ of $SU(6) \times U(1)$. The doublet-triplet splitting can be achieved through a Higgs superpotential that contains terms of the following form:

$$W_H \supset \mathbf{15}_H^0 \mathbf{15}_H^0 \mathbf{15}_H^0 + \mathbf{\bar{15}}_H^0 \mathbf{\bar{15}}_H^0 \mathbf{\bar{15}}_H^0 + M_1 (\mathbf{\bar{6}}_H^1 \mathbf{6}_H^{-1}) + (\mathbf{\bar{6}}_H^{-1} \mathbf{6}_H^1 - M_2^2) Z + (\mathbf{\bar{15}}_H^0 \mathbf{15}_H^0 - M_3^2) Z', \quad (2)$$

where Z and Z' are gauge singlets that are integrated out to give nonzero superlarge VEVs to the components that break $SU(6) \times U(1)$ to the Standard Models group at superlarge scales. There may be other terms in the Higgs superpotential as well. We do not write terms of the form $M(\mathbf{\bar{15}}_H^0 \mathbf{15}_H^0)$ and $M(\mathbf{\bar{6}}_H^{-1} \mathbf{6}_H^1)$ in Eq. (2), since such terms can be eliminated simply by shifting the singlet superfields Z and Z' . (Or one could forbid such terms with a discrete symmetry.)

The color-triplet Higgs(ino) fields in $\mathbf{15}_H^0$ and $\mathbf{\bar{15}}_H^0$ obtain mass from the first terms in Eq. (2), which contain $\epsilon_{12abc6} \langle H^{12} \rangle H^{bc} H^{a6} + \epsilon^{12abc6} \langle H_{12} \rangle H_{bc} H_{a6}$. The doublet-triplet splitting works essentially as shown in Eq. (1). There are altogether three triplet Higgs multiplets with Standard Model quantum numbers $(3, 1, -\frac{1}{3})$, which are $H^{a6} \in \mathbf{15}_H^0$, $H_{bc} \in \mathbf{\bar{15}}_H^0$, and $H^{(-1)a} \in \mathbf{6}_H^{-1}$; and there are three anti-triplet Higgs in $(\bar{3}, 1, +\frac{1}{3})$ that are conjugate to these. The mass matrix of these colored Higgs(ino) fields with $Y/2 = \pm\frac{1}{3}$ is

$$W_{(3,1,-1/3)} = \left(H^{a6}, \epsilon^{abc} H_{bc}, H^{(-1)a} \right) \begin{pmatrix} 0 & \langle H^{12} \rangle & 0 \\ \langle H_{12} \rangle & 0 & 0 \\ 0 & 0 & M_1 \end{pmatrix} \begin{pmatrix} H_{a6} \\ \epsilon_{abc} H^{bc} \\ H_a^{(1)} \end{pmatrix}. \quad (3)$$

All these triplets acquire superlarge mass. There are also a pair of triplets with Standard Model quantum numbers $(3, 1, \frac{2}{3}) + (\bar{3}, 1, -\frac{2}{3})$, namely $H^{(1)a} \in \mathbf{6}_H^1$ and $H_a^{(-1)} \in \bar{\mathbf{6}}_H^{-1}$. These obtain no mass from the terms in Eq. (2), and indeed are goldstone fields that get eaten by the Higgs mechanism when $H^{(1)6}$ and $H_6^{(-1)}$ get superlarge VEVs.

There are altogether three doublet Higgs multiplets with Standard Model quantum numbers $(1, 2, \frac{1}{2})$, which are $H_{i6} \in \bar{\mathbf{15}}_H^0$, $H^{(1)i} \in \mathbf{6}_H^1$, and $H_i^{(1)} \in \bar{\mathbf{6}}_H^1$; and there are three doublet Higgs with $(1, 2, -\frac{1}{2})$ that are conjugate representations to these. The mass matrix of these doublet Higgs is

$$W_{(1,2,1/2)} = \left(H_{i6}, H^{(1)i}, H_i^{(1)} \right) \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & M_1 \end{pmatrix} \begin{pmatrix} H^{i6} \\ H_i^{(-1)} \\ H^{(-1)i} \end{pmatrix}. \quad (4)$$

Two pairs of doublet Higgs fields are left massless by the mass matrix in Eq. (4). (This is due to the absence of the terms of the form $M(\bar{\mathbf{15}}_H^0 \mathbf{15}_H^0)$ and $M(\bar{\mathbf{6}}_H^{-1} \mathbf{6}_H^1)$ in Eq. (2), which was discussed above. That is to say, it is due to the fact that $\langle Z \rangle = 0$ and $\langle Z' \rangle = 0$.) Of these two massless pairs of doublets, one pair consists of goldstone fields that get eaten by the Higgs mechanism, and the other pair are the two doublet Higgs multiplets of the MSSM. The MSSM doublet with $(1, 2, \frac{1}{2})$ is a linear combination of the doublets in $\bar{\mathbf{15}}_H^0$ and $\mathbf{6}_H^1$, and the $(1, 2, -\frac{1}{2})$ doublet of the MSSM is a linear combination of the doublets in $\mathbf{15}_H^0$ and $\bar{\mathbf{6}}_H^{-1}$. Finally, there are also the goldstone modes $H^{ia} \in \mathbf{15}_H^0$ and $H_{ia} \in \bar{\mathbf{15}}_H^0$ that are eaten by the Higgs mechanism. The absence of uneaten goldstone Higgs fields implies that this minimum is not continuously connected to other degenerate minima, which is a condition for satisfactory breaking of $SU(6) \times U(1)$ to the Standard Model.

Although we arrived at this set-up by generalizing flipped $SU(5) \times U(1)$ to $SU(6) \times U(1)$, it may not be obvious from examining the mass matrices in Eqs. (3) and (4) that the doublet-triplet splitting is being accomplished here by the missing partner mechanism. It can be seen more readily if we

note that the $\mathbf{15}_H^0$ contains both a $(3, 1, -\frac{1}{3})$ (namely H^{a6}) and a $(\bar{3}, 1, +\frac{1}{3})$ (namely $\epsilon_{abc}H^{bc}$), and these are “partners” of each other in a mass term of the form $\epsilon_{12abc}H^{a6}H^{bc}\langle H^{12} \rangle$. On the other hand, in the $\mathbf{15}_H^0$ there is a $(1, 2, -\frac{1}{2})$ (namely H^{i6}), but no $(1, 2, +\frac{1}{2})$. That is, the “partner” of the doublet is “missing”. The same is true for the $\overline{\mathbf{15}}_H^0$. It is important that the light doublets in $\mathbf{15}_H^0$ and $\overline{\mathbf{15}}_H^0$ are not connected to each other by an explicit mass term $M(\mathbf{15}_H^0\overline{\mathbf{15}}_H^0)$. This is analogous to the condition in flipped $SU(5)$ that there is no term $M(\mathbf{10}_H\overline{\mathbf{10}}_H)$. The $\mathbf{15}_H^0$ and $\overline{\mathbf{15}}_H^0$ of $SU(6) \times U(1)$ contain the Higgs multiplets that do the doublet-triplet splitting in flipped $SU(5) \times U(1)$, namely $\mathbf{10}_H$, $\mathbf{5}_H$, $\overline{\mathbf{10}}_H$, and $\overline{\mathbf{5}}_H$. The $\mathbf{6}_H$ and $\overline{\mathbf{6}}_H$ multiplets are required to do the breaking of $SU(6) \times U(1)$ to $SU(5) \times U(1)$. These additional Higgs multiplets slightly complicate matters, but do not affect the basic missing partner mechanism. The $\mathbf{6}_H^{(-1)}$ and $\overline{\mathbf{6}}_H^{(1)}$ are connected by a mass term, and thereby provide “partners” for each other. The $\mathbf{6}_H^{(1)}$ and $\overline{\mathbf{6}}_H^{(-1)}$ are *not* connected to each other by a mass term (because $\langle Z \rangle = 0$), but that does not matter: the triplets in them are eaten and the extra pair of doublets in them is needed, since a pair of doublets must be eaten by the Higgs mechanism when $SU(6)$ breaks to $SU(5)$.

Turning now to the spectrum of quarks and leptons, the extra vectorlike quarks and leptons in $\mathbf{5}^{-2} + \overline{\mathbf{5}}^2$ (called “heavy” in Table I) obtain masses from the term $(\mathbf{15}^0\overline{\mathbf{6}}^1)\overline{\mathbf{6}}_H^{-1}$, which (in $SU(5) \times U(1)_X$ language) contains $(\mathbf{5}^{-2}\overline{\mathbf{5}}^2)\langle \mathbf{1}_H^0 \rangle$. The masses of the light quarks and leptons arise from the following terms:

$$\begin{aligned}
M_D : (\mathbf{15}^0\mathbf{15}^0)\mathbf{15}_H^0 & \supset (\mathbf{10}^1\mathbf{10}^1)\langle \mathbf{5}_H^{-2} \rangle \\
\supset \epsilon_{12abc}(\psi^{1a}\psi^{bc})\langle H^{26} \rangle & \supset \epsilon_{12abc}(\psi^{1a}\psi^{bc})\langle H^2 \rangle, \\
\\
M_L : (\overline{\mathbf{6}}^1\overline{\mathbf{6}}^{-1})\mathbf{15}_H^0 & \supset (\mathbf{1}^5\overline{\mathbf{5}}^{-3})\langle \mathbf{5}_H^{-2} \rangle \\
\supset (\psi_6^{(1)}\psi_2^{(-1)})\langle H^{26} \rangle & \supset (\psi\psi_2)\langle H^2 \rangle, \\
\\
M_U, M_N : (\mathbf{15}^0\overline{\mathbf{6}}^{-1})\overline{\mathbf{15}}_H^0\mathbf{6}_H^1/M & \supset (\mathbf{10}^1\overline{\mathbf{5}}^{-3})\langle \overline{\mathbf{5}}_H^2 \rangle\langle \mathbf{1}_H^0 \rangle/M \\
\supset (\psi^{2a}\psi_a^{(-1)} + \psi^{21}\psi_1^{(-1)})H_{26}H^{(1)6}/M & \supset (\psi^{2a}\psi_a + \psi^{21}\psi_1)\langle H_2 \rangle\langle H \rangle/M,
\end{aligned} \tag{5}$$

where M_D , M_L , M_U , and M_N are the mass matrices of the down quarks, charged leptons, up quarks, and neutrino Dirac mass matrix, respectively.

Note that unlike the usual flipped $SU(5)$ models, the mass matrix of the up quarks and the Dirac mass matrix of the neutrinos must come from a higher-dimension operator. (The reason is that the light Higgs doublets that break the weak interactions are purely in $\overline{\mathbf{15}}_H^0$ and $\mathbf{6}_H^1$ and not in $\overline{\mathbf{6}}_H^1$, so that the renormalizable Yukawa term $(\mathbf{15}^0 \overline{\mathbf{6}}^{-1}) \overline{\mathbf{6}}_H^1$ does not give mass to these fermions.) This higher-dimension operator can be obtained by integrating out an adjoint of superheavy quarks and leptons.

We now turn to the group $SU(7) \times U(1)$. There are several anomaly-free sets of $SU(7)$ multiplets of fermions that lead to three families at low energies. We will consider $\mathbf{21} + \overline{\mathbf{7}} + \overline{\mathbf{7}} + \overline{\mathbf{7}}$. Other cases work out in similar ways. There is only one solution for the $U(1)$ charge assignments (up to an arbitrary normalization) that is anomaly-free and family-independent, namely $\mathbf{21}^0 + \overline{\mathbf{7}}^1 + \overline{\mathbf{7}}^{-1} + \overline{\mathbf{7}}^0$. The simplest set of Higgs supermultiplets to break to the Standard Model and give mass to the quarks and leptons is $(\mathbf{35}_H^0 + \overline{\mathbf{35}}_H^0) + (\mathbf{21}_H^0 + \overline{\mathbf{21}}_H^0) + (\overline{\mathbf{7}}_H^0 + \mathbf{7}_H^0) + (\overline{\mathbf{7}}_H^1 + \mathbf{7}_H^{-1}) + (\overline{\mathbf{7}}_H^{-1} + \mathbf{7}_H^1)$.

The breaking to the Standard Model group can be done by the following Higgs vacuum expectation values: $\langle H_6^{(-1)} \rangle \in \overline{\mathbf{7}}_H^{-1}$, $\langle H_7^{(0)} \rangle \in \overline{\mathbf{7}}_H^0$, $\langle H^{12} \rangle \in \mathbf{21}_H^0$, and their conjugates $\langle H^{(1)6} \rangle \in \mathbf{7}_H^1$, $\langle H^{(0)7} \rangle \in \mathbf{7}_H^0$, $\langle H_{12} \rangle \in \overline{\mathbf{21}}_H^0$. Denote by T_1 the generator of the $U(1)$, and by T_7 the $SU(7)$ generator $\text{diag}(1, 1, 1, 1, 1, -5, 0)$. The VEVs $\langle H_7^{(0)} \rangle$ and $\langle H_6^{(-1)} \rangle$ break $SU(7) \times U(1)$ down to $SU(5) \times U(1)_X$, where $X = \frac{1}{2}(T_7 + 5T_1)$. The VEVs $\langle H^{12} \rangle$ and $\langle H_{12} \rangle$ do the “flipped” breaking, leaving unbroken $Y/2 = \frac{1}{5}(-Y_5/2 + X)$.

The quark-lepton and Higgs multiplets decompose under subgroups of $SU(7) \times U(1)$ as shown in Table III.

$SU(7)^{T_1}$	$SU(5)^{T_7, T_1}$	$SU(5)^X$	<i>quarks/leptons</i>	<i>Higgs fields</i>
$\mathbf{21}^0$	$\mathbf{10}^{2,0}$	$\mathbf{10}^1$	$\psi^{\alpha\beta} = \text{light}$	$\langle H^{12} \rangle \sim M_G$
	$\mathbf{5}^{-4,0}$	$\mathbf{5}^{-2}$	$\psi^{\alpha 6} = \text{heavy}$	$\langle H^{26} \rangle \sim M_W$
	$\mathbf{5}^{1,0}$	$\mathbf{5}^{1/2}$	$\psi^{\alpha 7} = \text{heavy}$	$\langle H^{\alpha 7} \rangle = 0$
	$\mathbf{1}^{-5,0}$	$\mathbf{1}^{-5/2}$	$\psi^{67} = \text{heavy}$	$\langle H^{67} \rangle = 0$
$\overline{\mathbf{7}}^0$	$\overline{\mathbf{5}}^{-1,0}$	$\overline{\mathbf{5}}^{-1/2}$	$\psi_\alpha^{(0)} = \text{heavy}$	$\langle H_\alpha^{(0)} \rangle = 0$
	$\mathbf{1}^{5,0}$	$\mathbf{1}^{5/2}$	$\psi_6^{(0)} = \text{heavy}$	$\langle H_6^{(0)} \rangle = 0$
	$\mathbf{1}^{0,0}$	$\mathbf{1}^0$	$\psi_7^{(0)} = \text{heavy}$	$\langle H_7^{(0)} \rangle \sim M_G$
$\overline{\mathbf{7}}^1$	$\overline{\mathbf{5}}^{-1,1}$	$\overline{\mathbf{5}}^2$	$\psi_\alpha^{(1)} = \text{heavy}$	$\langle H_\alpha^{(1)} \rangle = 0$
	$\mathbf{1}^{5,1}$	$\mathbf{1}^5$	$\psi_6^{(0)} = \text{light}$	$\langle H_6^{(1)} \rangle = 0$
	$\mathbf{1}^{0,1}$	$\mathbf{1}^{5/2}$	$\psi_7^{(0)} = \text{heavy}$	$\langle H_7^{(1)} \rangle = 0$
$\overline{\mathbf{7}}^{-1}$	$\overline{\mathbf{5}}^{-1,-1}$	$\overline{\mathbf{5}}^{-3}$	$\psi_\alpha^{(-1)} = \text{light}$	$\langle H_\alpha^{(-1)} \rangle = 0$
	$\mathbf{1}^{5,-1}$	$\mathbf{1}^0$	$\psi_6^{(-1)} = \text{heavy}$	$\langle H_6^{(-1)} \rangle \sim M_G$
	$\mathbf{1}^{0,-1}$	$\mathbf{1}^{-5/2}$	$\psi_7^{(-1)} = \text{heavy}$	$\langle H_7^{(-1)} \rangle = 0$
$\mathbf{35}^0$	$\overline{\mathbf{10}}^{3,0}$	$\overline{\mathbf{10}}^{3/2}$		$\langle H^{\alpha\beta\gamma} \rangle = 0$
	$\mathbf{10}^{-3,0}$	$\mathbf{10}^{-3/2}$		$\langle H^{\alpha\beta 6} \rangle = 0$
	$\mathbf{10}^{2,0}$	$\mathbf{10}^1$		$\langle H^{\alpha\beta 7} \rangle = 0$
	$\mathbf{5}^{-4,0}$	$\mathbf{5}^{-2}$		$\langle H^{\alpha 67} \rangle = 0$

Table III.

The doublets H_{i6} and $H_{i\bar{6}}$ are the light doublets of the MSSM and obtain weak-scale VEVs. How they remain light will be seen next.

The doublet-triplet splitting can be achieved through a Higgs superpotential that contains terms of the following form:

$$\begin{aligned}
W_H \supset & \mathbf{35}_H^0 \mathbf{21}_H^0 \mathbf{21}_H^0 + \overline{\mathbf{35}}_H^0 \overline{\mathbf{21}}_H^0 \overline{\mathbf{21}}_H^0 + \mathbf{21}_H^0 \overline{\mathbf{7}}_H^0 \overline{\mathbf{7}}_H^{0'} + \overline{\mathbf{21}}_H^0 \mathbf{7}_H^0 \mathbf{7}_H^{0'} \\
& + M_1 (\overline{\mathbf{7}}_H^1 \mathbf{7}_H^{-1}) + (\overline{\mathbf{7}}_H^{-1} \mathbf{7}_H^1 - M_2^2) Z \\
& + (\overline{\mathbf{21}}_H^0 \mathbf{21}_H^0 - M_3^2) Z' + M_4 (\overline{\mathbf{35}}_H^0 \mathbf{35}_H^0) + (\overline{\mathbf{7}}_H^0 \mathbf{7}_H^0 - M_5^2) Z''.
\end{aligned} \tag{6}$$

There may be other terms in the Higgs superpotential as well. We do not include in Eq. (6) terms of the form $M(\overline{\mathbf{21}}_H^0 \mathbf{21}_H^0)$ and $M(\overline{\mathbf{7}}_H^{-1} \mathbf{7}_H^1)$, as they

can be absorbed by a shift in the singlet superfields Z and Z' . Note that there has to be more than one pair of Higgs fields of type $\mathbf{7}_H^0 + \overline{\mathbf{7}}_H^0$ to have the couplings $\mathbf{21}_H^0 \overline{\mathbf{7}}_H^0 \overline{\mathbf{7}}_H^{0'} + \overline{\mathbf{21}}_H^0 \mathbf{7}_H^0 \mathbf{7}_H^{0'}$, since the $\mathbf{21}$ and $\overline{\mathbf{21}}$ are antisymmetric tensors. We assume that there are exactly two $\mathbf{7}_H^0$, which we distinguish as unprimed and primed, and similarly for their conjugates.

This is just the straightforward generalization of the $SU(6) \times U(1)$ case. It is easily shown that all of the doublet and triplet Higgs fields are either given superheavy mass by the terms in Eq. (6) or get eaten by the Higgs mechanism, except for a pair of light doublets, which play the role of the Higgs doublets of the MSSM. To see this we must first consider what fields get eaten. The gauge fields in the coset $SU(7)/SU(5)$ that gets broken include A_6^i , A_7^i , A_6^a , and A_7^a , which have Standard Model quantum numbers $(1, 2, \frac{1}{2})$, $(1, 2, 0)$, $(3, 1, \frac{2}{3})$, and $(3, 1, \frac{1}{6})$, respectively, as well as their conjugates. There must exist goldstone modes in these representations to get eaten.

The mass matrix of the colored Higgs(ino) fields that are in $(3, 1, -\frac{1}{3}) + conj.$ is

$$W_{(3,1,-1/3)} = \begin{pmatrix} H^{a67} \\ \epsilon^{abc} H_{bc7} \\ H^{a6} \\ \epsilon^{abc} H_{bc} \\ H^{(-1)a} \end{pmatrix}^T \begin{pmatrix} M_4 & 0 & 0 & \langle H^{12} \rangle & 0 \\ 0 & M_4 & \langle H_{12} \rangle & 0 & 0 \\ 0 & \langle H^{12} \rangle & 0 & 0 & 0 \\ \langle H_{12} \rangle & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & M_1 \end{pmatrix} \begin{pmatrix} H_{a67} \\ \epsilon_{abc} H^{bc7} \\ H_{a6} \\ \epsilon_{abc} H^{bc} \\ H_a^{(1)} \end{pmatrix}. \quad (7)$$

Note that this has no zero eigenvalues, which is consistent with the fact that no fields of this type eaten. All these fields get superheavy masses.

The mass matrix of the colored Higgs(ino) fields that are in $(3, 1, \frac{1}{6}) + conj.$ is

$$W_{(3,1,1/6)} = \begin{pmatrix} \epsilon^{abc} H_{bc6} \\ H^{a7} \\ H^{(0)a} \\ H^{(0)'a} \end{pmatrix}^T \begin{pmatrix} M_4 & \langle H_{12} \rangle & 0 & 0 \\ \langle H^{12} \rangle & 0 & \langle H_7^{(0)'} \rangle & \langle H_7^{(0)} \rangle \\ 0 & \langle H^{(0)'} \rangle & 0 & 0 \\ 0 & \langle H^{(0)'} \rangle & 0 & 0 \end{pmatrix} \begin{pmatrix} \epsilon_{ab'c'} H^{b'c'6} \\ H_{a7} \\ H_a^{(0)} \\ H^{(0)'a} \end{pmatrix}. \quad (8)$$

The vanishing of the lower-right 2-by-2 block is due to the absence of a mass term of the form $M(\overline{\mathbf{7}}_H^0 \mathbf{7}_H^0)$ (or, equivalently, the vanishing of the VEV of

Z''). There is one zero eigenvalue of this matrix, corresponding to a pair of massless multiplets with $Y/2 = \pm\frac{1}{6}$ that gets eaten. All the other fields of this type get superheavy mass.

There is finally a single pair of Higgs(ino) fields in $(3, 1, \frac{2}{3}) + conj.$, namely $H^{(1)a}$ and $H_a^{(-1)}$. These have no mass term, since there is no mass term of the form $M(\bar{\mathbf{7}}_H^{-1} \mathbf{7}_H^1)$ (or, equivalently, the VEV of Z vanishes). These are goldstone modes that get eaten. The mass matrix of the doublet Higgs(ino) fields that are in $(1, 2, \frac{1}{2}) + conj.$ is

$$W_{(1,2,1/2)} = \begin{pmatrix} H_{i67} \\ H_{i6} \\ H_i^{(1)} \\ H^{(1)i} \end{pmatrix}^T \begin{pmatrix} M_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & M_1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} H^{i67} \\ H^{i6} \\ H^{(-1)i} \\ H_i^{(-1)} \end{pmatrix}. \quad (9)$$

There are two zero eigenvalues, corresponding to two pairs of massless multiplets with $Y/2 = \pm\frac{1}{2}$. One of these pairs is eaten, as we have seen above. This leads to a single massless pair of doublets that are the doublet Higgs(ino) fields of the MSSM.

Finally, there are Higgs(ino) fields in $(1, 2, 0)$. These have a mass matrix

$$W_{(1,2,0)} = \begin{pmatrix} H^{i7} \\ H^{(0)i} \\ H^{(0)'i} \end{pmatrix}^T \begin{pmatrix} 0 & \langle H_7^{(0)'} \rangle & \langle H_7^{(0)} \rangle \\ \langle H^{(0)'}7 \rangle & 0 & 0 \\ \langle H^{(0)}7 \rangle & 0 & 0 \end{pmatrix} \begin{pmatrix} H_{i7} \\ H_7^{(0)} \\ H_i^{(0)'} \end{pmatrix}. \quad (10)$$

here there is one zero eigenvalue, corresponding to the pair of massless multiplets that gets eaten. All other fields of this type get superheavy mass.

In sum, the only Higgs(ino) fields that do not get eaten or get superheavy mass are the pair of doublets of the MSSM. In other words, there is successful doublet-triplet splitting.

We have shown that the missing partner mechanism can be implemented in larger unitary gauge groups, by a straightforward generalization of the well-known implementation in flipped $SU(5) \times U(1)$. The details are more involved, due to the larger groups and the correspondingly larger multiplets. But the essential idea is no more complicated than in flipped $SU(5)$. The examples that have been worked out in detail here show that the generalization to unitary groups larger than $SU(5)$ does not require complicated conditions to be imposed on the superpotential or the structure of the theory.

References

- [1] H. Georgi, *Nucl. Phys.* **B156**, 126 (1982).
- [2] S.M. Barr, *Phys. Rev.* **D21**, 1424 (1980); *ibid.* **D78**, 055008 (2008); *ibid.* **D78**, 075001 (2008).
- [3] L. Maiani, in *Comptes Rendus de l'Ecole d'Ete de Physiques des Particules*, Gif-sur-Yvette, 1979, IN2P3, Paris, p.3; S. Dimopoulos and H. Georgi, *Nucl. Phys.* **B150**, 193 (1981); M. Sakai, *Z. Phys.* **B188**, 573 (1981).
- [4] H. Georgi, *Phys. Lett.* **108B**, 282 (1982); A. Masiero, D.V. Nanopoulos, K. Tamvakis, and T. Yanagida, *Phys. Lett.* **115B**, 380 (1982); B. Grinstein, *Nucl. Phys.* **B206**, 387 (1982).
- [5] E. Witten, *Phys. Lett.* **105B**, 267 (1981); D.V. Nanopoulos and K. Tamvakis, *Phys. Lett.* **113B**, 151 (1982); S. Dimopoulos and H. Georgi, *Phys. Lett.* **117B**, 287 (1982); L. Ibanez and G. Ross, *Phys. Lett.* **110B**, 215 (1982).
- [6] S.M. Barr, *Phys. Rev.* **D57**, 190 (1998).
- [7] K. Inoue, A. Kakuto, and T. Takano, *Prog. Theor. Phys.* **75**, 664 (1986); A. Anselm and A. Johansen, *Phys. Lett.* **200B**, 331 (1988); Z.G. Berezhiani, *Phys. Lett.* **355B**, 481 (1995).
- [8] I. Antoniadis, J. Ellis, J. Hagelin, and D.V. Nanopoulos, *Phys. Lett.* **194B**, 231 (1987); *ibid.* **205B**, 455 (1988).